PACKING WORDS

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What is the minimum rectangular area in which one can pack precisely N words of length L? All must be joined in criss-cross fashion, and no words of other lengths are allowed. The main point is to find the packing patterns; finding real (and, if possible, common) words to fit those patterns adds interest, converting a purely mathematical problem to a logological one as well. The patterns derived here may be used in conventional crossword or other puzzles.

Packing patterns follow recognizable sequences as N increases. There are two types, depending on whether L is odd or even. This article describes packing four-letter and five-letter words; plans for six-letter or seven-letter words can be easily derived from them. I do not give a full scheme for six-letter words, but exhibit one large arrangement. For all word sizes, the examples given here are essentially portions of ordinary double word squares strung together.

Consider first four-letter words (L=4). In the following examples there is a fundamental pattern for five words (N=5). This is not the best possible packing, but it is the best which can be logically extended to larger values of N. Adding blocks extends it to 12, 19, 26 (not shown), etc. There is another fundamental pattern for N=6, to which one can add a word at a time to allow packing 7 or 8. For N=9, one must start to build a new block.

```
N MAIN   N MAIN MAIN   N MAIN MAIN MAIN
O E   O  OVER O E   O  OVER OVER O E   O
N A   N  N AMEN A   N  N AMEN AMEN A   N
EATS E   EATS EATS E   EATS EATS EATS E
5        12             19

  MAIN   N MAIN   N MAIN   T EXIT     T EXIT
OVER     OVER     OVER O   HEAR H     HEAR HEAR
  AMEN   N AMEN   N AMEN   E RARE     E RARE
EATS     EATS     EATS E   MANY MANY  MANY MANY
6        7        8        9          10

T EXIT E      EXIT EXIT   T EXIT EXIT
HEAR HEAR   HEAR HEAR     HEAR HEAR H
E RARE R      RARE RARE   E RARE RARE
MANY MANY   MANY MANY     MANY MANY M
11          13            15
```

All of the above (Set A) are four units high. The patterns on the next page (Set B) are five units high.

```
  R  F     R       R  F     R       R       R  F
B A L L   B A L L   B A L L   B A L L   B A L L   B A L L
A R E A   A R E A   A R E A   A R E A   A R E A   A R E A
N E S T   N E S T   N E S T   N E S T   N E S T   N E S T
D  S      D  S E N D  S      D  S E N D  S E N D  S
7         15                23
```

```
  R A F T     R       R A F T     R       R       R A F T
B A L L       B A L L   B A L L     B A L L   B A L L   B A L L
A R E A       A R E A   A R E A     A R E A   A R E A   A R E A
N E S T       N E S T   N E S T     N E S T   N E S T   N E S T
D             D  S E N D          D  S E N D  S E N D
8             16                  24
```

```
  R             R       R             R       R       R
B A L L   B     B A L L   B A L L   B     B A L L   B A L L   B A L L   B
A R E A   A     A R E A   A R E A   A     A R E A   A R E A   A R E A   A
N E S T   N     N E S T   N E S T   N     N E S T   N E S T   N E S T   N
D  S E N D      D  S E N D  S E N D      D  S E N D  S E N D  S E N D
9               17                      25
```

The table below summarizes the areas needed to pack N four-letter words for either Set A or Set B; the final line lists the smallest area allowed by any packing. Packing 31 words in 5x19 is the last case in which a five-high frame is useful. There are three special cases. Eight works pack in a normal 4x4 square; the scheme for seven words in Set B can be rotated into a four-high pattern; and for 26 words, packing in a 9x9 square (described below) beats either set. The final sequence could never serve as an IQ test unless you see how it was made!

words	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23
areaA	4	16	16	16	24	24	(24)	24	24	36	36	44	44	44	44	56	56	56	64	64	64	64	76
areaB							20	25	30						45	50	55						70
best	4	16	16	16	24	24	20	(16)	30	36	36	44	44	44	44	50	55	56	64	64	64	64	70

words	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41
areaA	76	76	84	84	84	84	96	96	96	104	104	104	104	116	116	116	124	124
areaB	75	80						95	100	110						120	125	130
best	75	76	(81)	84	84	84	96	95	96	104	104	104	104	116	116	116	124	124

Frames more than five units high usually result in a loss of packing efficiency, but one can get neat constructions in square envelopes. The word pattern in the 14x14 envelope at the top of the next page is made by the use of two basic units, a solid and a hollow block. The 9x9 array to the right uses the same blocks in a similar pattern; it contains 24 words. The third array is less symmetrical but contains 26 words. As already noted, this uses three fewer cells to pack 26 words than does the Area A construction.

All diagrams shown in this article use common words. The 4x4 square on the next page appears ordinary, but it is special in that it allows extending the central words in all directions into eight other words. By going to the computer for a special search, one can find squares with wrap-around properties that allow one square to be used throughout an array. Limiting the database

```
SCAB SITE SCAB
H RENT  ACHE E
IDEA E  S IDEA
PEAR PRAY PEAR
 L    A    S
SITE SCAB SITE
T  ACHE E T  A
E  S I EASE  S
PRAY PEAR PRAY
 A    S    J
SCAB SITE SCAB
HERE T  A HERE
I EASE  SAID A
PEAR PRAY PEAR
56
```

```
SCAB SITE
H RENT  A
IDEA E  S
PEAR PRAY
 L    J
SITE SCAB
T  A HERE
E  SAID A
PRAY PEAR
24
```

```
SCAB SIGN  SCAB
H RENT R   HERE
IDEA EDIT  IDEA
PEAR WINO  PEAR
 L    S M
SITE SCAB
I ITCH R
ZINC IDEA
E SHOP A
26
```

to 2500 common words allowed only a few squares like the ones below. The wrap-around sequences ALAREAL, RESTIRE, ALTORAL and MAIMAMA are reminiscent of directed word chains.

```
CAME CAME CAME
A AREA  REAL R
STIR S  R STIR
TOMS TOMS TOMS
  R    A    A
CAME CAME CAME
A  R A AREA  R
S  REST R S  R
TOMS TOMS TOMS
  R    R    A
CAME CAME CAME
ALAR A  R ALAR
S IRES  REST R
TOMS TOMS TOMS
56
```

```
BOSS BOSS
U TOFU  O
STEW S  W
TOMS TOMS
 R    U
BOSS BOSS
U  O UNTO
S  WEST W
TOMS TOMS
24
```

```
CAME  BOSS
ALAR  UNTO
STIR  STEW
TOMS  TOMS
```

Various plans for packing five-letter words are used in the following. One word can be removed at a time from the 15-word array to allow 14, 13, 12 or 11 words. No smaller rectangle containing precisely 11 or 10 words is possible. One or two words can be removed from the 19-word array, but three words cannot, so packing 16 five-letter words requires inefficient appending to the 15-word array.

```
MAPLE   MAPLE M
A H N   A H N A
SCOPE   S OPENS
T T M   T T M T
STORY   STORY S
6       7
```

```
MAPLE   MAPLE MAPLE
ASHEN   A HENNA H N
SCOPE   SCOPE SCOPE
TOTEM   TOTEM M T M
STORY   STORY STORY
10      15
```

```
CLIMB   CLIMB C
HONOR   H NORTH
AWARE   AWARE A
RENEW   R NEWER
TRESS   TRESS T
```

```
CLIMB CLIMB C
H NORTH NORTH
AWARE AWARE A
R NEWER NEWER
TRESS TRESS T
```

Just as four-letter words can be packed in frames five units high, so five-letter words can be packed in frames six units high. As before, determination of when L+1 is better than L is a matter of arithmetic. A summary is given in the table below. There are two exceptions (in parentheses), the normal 5x5 square and a case (described below) where a large square array allows an improved packing.

```
 A  B       A        A  B       A        A        A  B
ABOUT     ABOUT    ABOUT     ABOUT    ABOUT    ABOUT
FORTH     FORTH    FORTH     FORTH    FORTH    FORTH
ORATE     ORATE    ORATE     ORATE    ORATE    ORATE
OTTER     OTTER    OTTER     OTTER    OTTER    OTTER
T E E     T ERECT  E E       T ERECT  ERECT    E E
9         19                 29

 GAILY     A        CADGE      E        S        ABHOR
GARNI     ABACK    CABAL      DACHA    FACES    ABLER
RUSES     BORON    AROMA      ETHOS    ATOLL    ROUND
ADEPT     BUGLE    BOOES      FEINT    KYRIE    DUNCE
CYSTS     ETUDE    BLESS      ENVOI    IRATE    OTTER
E         Y ESSAY             R ERROR  LEPER
10        20                  30

 Y        GOATS      Y        Y        GOATS
FABLE     G U E      FABLE    FABLE    G U E
ACRID     ULTRA      ACRID    ACRID    ULTRA
THING     S E T      THING    THING    S E T
STAGE     TYRES      STAGE    STAGE    TYRES
O RODEO              O RODEO  RODEO
16                   26

 Y                    Y        Y
FABLE     F B E      FABLE    FABLE    F B E
ACRID     ACRID      ACRID    ACRID    ACRID
THING     T I G      THING    THING    T I G
STAGE     STAGE      STAGE    STAGE    STAGE
O RODEO   R D        O RODEO  RODEO    R D
15                   25
```

words	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28
areaA	35	35	35	55	55	55	55	55	75	65	65	65	85	85	85	85	85	100	95	95	95
areaB		30	36					66	72			66	72					102	108		
best	35	30(25)	55	55	55	55	55	72	65	65	65	72	85	85	85	85	100	95	95	95	

words	29	30	31	32	33	34	35	36	37	38	39	40
areaA	115	115	115	115	115	135	125	125	125	145	145	145
areaB	102	108					138	144			138	144
best	102	108	115	115	115	(121)	125	125	125	145	138	144

Packing in large squares is more efficient for five-letter words than for four-letter words. Two criss-cross patterns are illustrated on the next page. A 5x5 double word square places ten words in 25 cells for 0.400 words per cell; the two squares illustrated pack at 0.282 words per cell. They were developed by joining 5x5 squares found by computer; in some places the reader can recognize the original square by inserting the letter(s)

deleted in order to make the joins.

```
CABLE KVASS OMENS    KNACK FARAD
A LOWER V LEVEL M    N TROLL A A
SLAVE ALIBI ETUDE    IGLOO EAVED
EASER ALARM NADIR    F ANKLE E D
DRESS LANES SLEEK    EASES TODAY
 G     N A     T      V     P R
DEALS GOADS FEAST    LASTS HEWED
A RIATA U UHLAN R    ESTOP ARRAY
WHETS UNDER ATONE    ATONE RAISE
N NESTS I FAMED A    S RENTS T R
SCARY SPOTS ERECT    TRYST HEEDS
 O     A H     O     34
SPORT ASPIC GNOME
LEMUR PHONO RUBES
ENEMY RAKED ADITS
W GESSO E ADMIT E
SLANT NERDS PESOS
82
```

Packing six-letters words follows the general plan used for four-letter ones; the reader may enjoy working out the arithmetic. In the six-letter word square below, packing efficiency approaches that of ordinary crosswords. Packing seven-letter words is even more efficient, but finding actual words is difficult; BASIC is too slow for finding 7x7 double squares. I leave the task of filling in the array below to programmers like Ted Clarke.

```
CABALA EAGLES DABBED    AAAAAAA AAAAAAA AAAAAAA
A ACUMEN  I PREVUE U    A AAAAAAA A A AAAAAAA A
PERUSE TORQUE FERRIC    AAAAAAA AAAAAAA AAAAAAA
OLEATE A  U A ARDENT    AAAAAAA AAAAAAA AAAAAAA
TASTER I  O R TSETSE    AAAAAAA AAAAAAA AAAAAAA
ENTERS LITRES SENSED    AAAAAAA AAAAAAA AAAAAAA
 D       L       C     AAAAAAA AAAAAAA AAAAAAA
ISLAND BIASED SWEATY      A       A A A       A
M I  E E CAVORT  M A    AAAAAAA AAAAAAA AAAAAAA
A N  J ABATED EYEBAR    A AAAAAAA A A AAAAAAA A
GREASE RACING R  U R    AAAAAAA AAAAAAA AAAAAAA
E A  CRETIN E I  S O    A AAAAAAA A A AAAAAAA A
SPROUT ROASTS CASHEW    AAAAAAA AAAAAAA AAAAAAA
 I      N       V      A AAAAAAA A A AAAAAAA A
MATTER PSALMS FABLED    AAAAAAA AAAAAAA AAAAAAA
ANYONE I L  H IGUANA      A       A A A       A
COMPTS R E  I LETTER    AAAAAAA AAAAAAA AAAAAAA
ASPIRE AUXINS INTERN    AAAAAAA AAAAAAA AAAAAAA
W ACETIC I  TRADES E    AAAAAAA AAAAAAA AAAAAAA
SENSES YEARNS LASTED    AAAAAAA AAAAAAA AAAAAAA
                        AAAAAAA AAAAAAA AAAAAAA
                        A AAAAAAA A A AAAAAAA A
                        AAAAAAA AAAAAAA AAAAAAA
```